

Simple model for transport phenomena : Microscopic construction of Maxwell Demon like engine

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We present a microscopic Hamiltonian framework to develop Maxwell demon like engine. Our model consists of a equilibrium thermal bath and a non-equilibrium bath; latter generated by driving with an external stationary, Gaussian noise. The engine we develop, can be considered as a device to extract work by modifying internal fluctuations. Our theoretical analysis focusses on finding the essential ingredients necessary for generating fluctuation induced transport under non-equilibrium condition. An important outcome of our model is that the net motion occurs when the non-linear bath is modulated by the external noise, creating the non-zero effective temperature even when the temperature of both the baths are same.

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I. INTRODUCTION

It is well known from elementary text books of thermodynamics that it is possible to extract some amount of mechanical work from a thermal bath at a temperature \bar{T} provided there is another bath at a lower temperature T ($\bar{T} > T$). Thermal engines are the devices that perform this task. We also know that any object in a thermal bath follows principle of equipartition of energy and exhibits random energy fluctuations of the order of $k_B T$. In the macroscopic scale this fluctuations are very small but of very import relevance for nanoscopic objects such as biological motors¹. The question one can raise, if it is possible to rectify thermal fluctuations by some appropriate mechanical devices, e.g., Maxwell's demon like engine or Carnot engine. Maxwell's demon manages to decrease the entropy, in other words, it increases the amount of energy available by increasing its knowledge about the motion of all the molecules. Thermodynamics says this is impossible, one can only increase entropy. To resolve this paradox (violation of second law of thermodynamics), an inchoate relationship between information and energy emerged. This problem has led to very interesting links between physics, information theory and the theory of computation - from concepts of information entropy to reversible computing. Feynman used a ratchet example to illustrate some implications of the second law of thermodynamics² and now it is quite well known that useful work cannot be extracted from systems undergoing equilibrium fluctuations. Under thermal equilibrium situation, no net particle current can be generated in the presence of external potential of arbitrary shape in accordance with the principle of detail balance. On the other hand, in non-equilibrium situation, due to the breakdown of the principle of detail balance net current flow is possible and thus one can transform energy into work at the expense of increased entropy³. The study of such a process from a microscopic point of view is quite chal-

lenging and hence an important topic in numerous areas of molecular sciences because no principles of generality that hold for equilibrium cases are applicable in non-equilibrium situations^{4,5}. To explain such processes from theoretical point of view, various types of models have been proposed in the literature^{2,3,4,5}. Though most of them are phenomenological, models from first principle have also been proposed in this direction^{6,7}.

Brownian motion cannot create a steady flux in a system in equilibrium. Nor can local asymmetries in a static potential energy landscape (simple example of non-equilibrium situation) rectify Brownian motion to induce a drift. A landscape that varies in time, however, can eke a flux out of random fluctuations by breaking spatiotemporal symmetry. Such flux-inducing time dependent potentials are known as thermal ratchets^{3,5,8}, and their ability to bias diffusion by rectifying thermal fluctuations has been proposed as a possible mechanism for transport by molecular motors and is being actively exploited for macromolecular separation^{9,10}.

Most thermal ratchet models are based on spatially asymmetric potentials. Their time variation involves displacing or tilting them relative to the laboratory frame, modulating their amplitude, changing their periodicity, or some combination, usually in a two-state cycle. It is now well known that a spatially symmetric potential still can induce drift in a cycle of three states, one of which allows for free diffusion. More recently, directed transport has been induced in an atomic cloud by a spatially symmetric rocking ratchet created with an optical lattice¹¹.

In this paper we investigate a self-consistent fluctuation induced transport theory through a microscopically constructed Maxwell demon type information engine. In our theory, the system under consideration is coupled to two independent baths maintained at two different equilibrium temperatures. A thermal fluctuation in one bath is created by modulating it externally via a random force, there by making the whole system thermodynamically open under the exposure of non-equilibrium fluctuations.

We then derive a mathematical expression for the fluctuation induced transport current under non-equilibrium situation analytically, which holds for all temperatures and then apply it to various cases of physical relevance, particularly, in the calculation of escape rate.

The prime advantage of adopting the external noise driven non-equilibrium bath in the present formalism is as follows. The external noise drives the heat bath out of equilibrium; as a result, a shift in the equilibrium temperature takes place through the creation of an effective temperature which the system of interest experiences in the steady state^{12,13}. Creation of non-equilibrium state (and of an effective temperature) through external driving is one of the essential requirements to break the symmetry of the system of interest that may lead to the generation of noise induced transport. In support of our definition of effective temperature, Popov and Hernandez¹⁴ recently provided an extensive and elegant analysis for defining generalized temperature in the context of non-equilibrium open systems.

A number of different situations depicting the modulation of one bath out of two, may be of physically relevant. As for example, we may think about heat transfer through a metallic rod when two ends of the rod are immersed in two (different) liquids kept at different equilibrium temperatures. The liquids may act as Brownian bath and the one which is photochemically active, may be exposed to an external fluctuating light intensity. Since the fluctuations in the light intensity result in the fluctuations in the polarization of the liquid molecules, the effective temperature field around the end of the metal bar gets modified, by making the liquids and rod system thermodynamically open and by throwing it in a non-equilibrium situation.

The organization of the paper is the following. In Sec.II we describe the construction and essential features of the microscopic, Hamiltonian based model. Analysis for external noise induced transport has been described in Sec.III. Sec.IV provides a general analysis for noise induced transport. The paper is concluded in Sec.V.

II. THE MODEL

In our model the system is coupled with two baths $A\{q_j, p_j\}$ and $B\{Q_j, P_j\}$ with characteristic frequencies $\{\omega_j\}$ and $\{\Omega_j\}$, respectively. The coupling between the system and the bath A and bath B is linear and nonlinear in nature, respectively. The Hamiltonian for the composite system can be written as¹⁵

$$\begin{aligned} H &= H_S + H_A + H_{SA} + H_B + H_{SB} + H_{int} \\ &= \frac{p^2}{2M} + U(x) + \frac{1}{2} \sum_{i=1}^N \left\{ \frac{p_i^2}{m_i} + m_i \omega_i^2 (q_i - g_i x)^2 \right\} \\ &\quad + \frac{1}{2} \sum_{j=1}^N \left\{ \frac{P_j^2}{M_j} + M_j \Omega_j^2 (Q_j - c_j f(x))^2 \right\} + H_{int}(1) \end{aligned}$$

with $H_{int} = \sum_{j=1}^N \kappa_j Q_j \epsilon(t)$. The first two terms on the right hand side of Eq. (1) represent the system mode and the third term describes the Hamiltonian of the bath A , maintaining the thermal environment of the engine, and the system bath interaction ($H_A + H_{SA}$). The fourth term corresponds to the bath B to which the system is nonlinearly coupled. The bath B is modulated by an external noise $\epsilon(t)$. H_{int} represents the interaction between the nonlinear modes Q_j and the external noise $\epsilon(t)$. In Eq.(1) $f(x)$ is some smooth function of the system variable x while g_i , c_j and κ_j are the coupling constants between the system and bath A , the system and bath B and the bath B and the external driving force $\epsilon(t)$, respectively. Both the baths are in thermal equilibrium with characteristic temperature T_a (for bath A) and T_b (for bath B), respectively, in presence of the system. In comparison to the earlier developments^{6,7}, in our model, the bath B is externally driven by an stationary gaussian noise agency. The external noise $\epsilon(t)$ has zero mean and arbitrary decaying correlation function:

$$\langle \epsilon(t) \rangle = 0, \langle \epsilon(t) \epsilon(t') \rangle = 2D\psi(t-t') \quad (2)$$

where D is the external noise strength and $\psi(t)$ is the memory kernel of external noise $\epsilon(t)$. The physical situation we address here is that at $t = 0$, both the baths A and B are in thermal equilibrium in the presence of the system but in the absence of the external noise agency. At $t = 0_+$, the external noise agency is switched on and the B -bath is modulated by $\epsilon(t)$.

After eliminating the bath variables, the equations of motion describing the system dynamics (considering the masses of the system and reservoir mode to be unity) is written as

$$\begin{aligned} \dot{x} &= v \\ \dot{v} &= -\frac{dU}{dx} - \int_0^t dt' \gamma_a(t-t')v(t') + \xi_a(t) \\ &\quad - \frac{df}{dx} \int_0^t dt' \gamma_b(t-t') \frac{df(x(t'))}{dx(t')} v(t') + \frac{df}{dx} \{ \xi_b(t) + \pi(t) \} \end{aligned} \quad (3)$$

where

$$\begin{aligned} \gamma_a(t) &= \sum_{i=1}^N g_i^2 \omega_i^2 \cos \omega_i t, \quad \gamma_b(t) = \sum_{j=1}^N c_j^2 \Omega_j^2 \cos \Omega_j t \\ \pi(t) &= - \int_0^t dt' \varphi(t-t') \epsilon(t'), \quad \varphi(t) = \sum_j c_j \Omega_j \kappa_j \sin(\Omega_j t) \\ \xi_a(t) &= \sum_{i=1}^N g_i \{ [q_i(0) - g_i x(0)] \omega_i^2 \cos \omega_i t + v_i(0) \omega_i \sin(\omega_i t) \} \\ \xi_b(t) &= \sum_{j=1}^N c_j \{ [Q_j(0) - c_j f(x(0))] \Omega_j^2 \cos \Omega_j t \\ &\quad + V_j(0) \Omega_j \sin(\Omega_j t) \} \end{aligned} \quad (4)$$

Here γ_a and γ_b are friction coefficient, generated due to coupling of the system with the two baths. $\{q_i(0), v_i(0)\}$

and $\{Q_j(0), V_j(0)\}$ are the initial values of the two bath variables. $\xi_a(t)$ and $\xi_b(t)$ are two noises due to the presence of the two baths A and B , respectively. The statistical properties of $\xi_a(t)$ and $\xi_b(t)$ are found to be

$$\begin{aligned}\langle \xi_a(t) \rangle_a &= 0, \quad \langle \xi_a(t) \xi_a(t') \rangle_a = k_B T_a \gamma_a(t - t') \\ \langle \xi_b(t) \rangle_b &= 0, \quad \langle \xi_b(t) \xi_b(t') \rangle_b = k_B T_b \gamma_b(t - t')\end{aligned}\quad (5)$$

where $\langle \dots \rangle_a$ and $\langle \dots \rangle_b$ are ensemble averages over the distributions of initial bath variables (for baths A and B , respectively) which are assumed to be canonically distributed with distribution functions:

$$\text{Bath A : } P_a = \frac{1}{Z_a} \exp \left(-\frac{H_A^0 + H_{SA}^0}{k_B T_a} \right) \quad (6a)$$

$$\text{Bath B : } P_b = \frac{1}{Z_b} \exp \left(-\frac{H_B^0 + H_{SB}^0}{k_B T_b} \right) \quad (6b)$$

with Z_a and Z_b being the two normalization constants and the superscript '0' in the Hamiltonian signifies the bath coordinates at time $t = 0$.

Let us now define an effective noise:

$$\eta(t) = \xi_b(t) + \pi(t). \quad (7)$$

As $\xi_b(t)$ and $\pi(t)$ are both stationary and Gaussian, the effective noise $\eta(t)$ will also be stationary and Gaussian. The statistical properties of $\eta(t)$ is given by

$$\begin{aligned}\langle \langle \eta(t) \rangle \rangle_b &= 0 \\ \langle \langle \eta(t) \eta(t') \rangle \rangle_b &= k_B T_a \gamma_a(t - t') + 2D \int_0^t dt'' \int_0^{t'} dt''' \\ &\quad \times \varphi(t - t'') \varphi(t' - t''') \psi(t'' - t''')\end{aligned}\quad (8)$$

where $\langle \langle \dots \rangle \rangle_b$ means we have taken two averages, averages of bath (B) variables and averages over each realization of $\epsilon(t)$, independently. It is important to mention the fact that the last equation, although has a close kinship with the famous fluctuation-dissipation relation, it is not the fluctuation-dissipation relation due to the presence of the external noise $\epsilon(t)$ through $\varphi(t)$. Rather, it serves as a thermodynamic consistency condition¹³.

To obtain finite result in the continuum limit ($N \rightarrow \infty$), we replace the summation by integration and consider density of modes $\mathcal{D}_a(\omega)$ and $\mathcal{D}_b(\Omega)$ for two baths A and B , respectively and assume the coupling functions as¹³:

$$g(\omega) = \frac{g_0}{\omega \sqrt{\tau_a}}, c(\Omega) = \frac{c_0}{\Omega \sqrt{\tau_b}} \text{ and } \kappa(\Omega) = \kappa_0 \Omega \sqrt{\tau_b} \quad (9)$$

where g_0 , c_0 and κ_0 are some constants. With these choices, $\gamma_a(t)$, $\gamma_b(t)$ and $\varphi(t)$ reduces to

$$\gamma_a(t) = \frac{g_0^2}{\tau_a} \int d\omega \mathcal{D}_a(\omega) \cos(\omega t), \quad (10a)$$

$$\gamma_b(t) = \frac{c_0^2}{\tau_b} \int d\Omega \mathcal{D}_b(\Omega) \cos \Omega t, \quad (10b)$$

$$\varphi(t) = c_0 \kappa_0 \int d\Omega \mathcal{D}_b(\Omega) \Omega \sin(\Omega t). \quad (10c)$$

We choose Lorentzian density of modes $\mathcal{D}_a(\omega)$ and $\mathcal{D}_b(\Omega)$:

$$\mathcal{D}_a(\omega) = \frac{2}{\pi} \frac{\tau_a}{1 + \omega^2 \tau_a^2} \text{ and } \mathcal{D}_b(\Omega) = \frac{2}{\pi} \frac{\tau_b}{1 + \Omega^2 \tau_b^2}. \quad (11)$$

These choices resemble broadly the behavior of the hydrodynamical modes in a macroscopic system¹⁶. With these forms of $\mathcal{D}_a(\omega)$, $\mathcal{D}_b(\Omega)$, $g(\omega)$, $c(\omega)$ and $\kappa(\omega)$ we have the expression for $\gamma_a(t)$, $\gamma_b(t)$ and $\varphi(t)$ as

$$\gamma_a(t) = (\gamma_a / \tau_a) \exp(-t / \tau_a), \quad (12a)$$

$$\gamma_b(t) = (\gamma_b / \tau_b) \exp(-t / \tau_b), \quad (12b)$$

$$\varphi(t) = (c_0 \kappa_0 / \tau_b) \exp(-t / \tau_b) \quad (12c)$$

where $\gamma_a = g_0^2$ and $\gamma_b = c_0^2$. When the correlation times of the two baths, τ_a (for bath A) and τ_b (for bath B) both tend to zero, one obtains a δ -correlated noise process. Clearly for Markovian internal dissipation [$\tau_a \rightarrow 0$, $\tau_b \rightarrow 0$] one has $\gamma_a(t) = 2\gamma_a \delta(t)$, $\gamma_b(t) = 2\gamma_b \delta(t)$ and $\varphi(t) = 2c_0 \kappa_0 \delta(t)$.

III. GENERIC EXPRESSION FOR NOISE INDUCED TRANSPORT

A. Bath modulation by external white noise

At this point, we consider a specific statistical property of the external noise $\epsilon(t)$ which is considered to be Gaussian, stationary and δ -correlated noise with strength $D_0^{1/2}$:

$$\langle \epsilon(t) \rangle = 0, \quad \langle \epsilon(t) \epsilon(t') \rangle = 2D_0 \delta(t - t'). \quad (13)$$

Then the property of the *dressed noise* $\pi(t)$ can be written as

$$\langle \pi(t) \rangle = 0, \quad \langle \pi(t) \pi(t') \rangle = 2D_0 \gamma_b \kappa_0^2 \delta(t - t'). \quad (14)$$

and correspondingly the Langevin equation (3) becomes

$$\dot{x} = v, \quad \dot{v} = -\frac{dU}{dx} - \Gamma(x)v + \xi_a(t) + f'(x)\eta(t) \quad (15)$$

where

$$\begin{aligned}\Gamma(x) &= \gamma_a + \gamma_b [f'(x)]^2, \\ \langle \xi_a(t) \rangle &= 0, \quad \langle \xi_a(t) \xi_a(t') \rangle = 2\gamma_a k_B T_a \delta(t - t'), \\ \langle \langle \eta(t) \rangle \rangle_b &= 0, \quad \langle \langle \eta(t) \eta(t') \rangle \rangle_b = 2\gamma_b [k_B T_b + D_0 \kappa_0^2] \delta(t - t').\end{aligned}$$

The Langevin equation (15) describes a multiplicative noise process with space dependent dissipation.

For the case of large dissipation, one eliminates the fast variables adiabatically to get a simpler description of the system which is valid in much slower time scale. When the Brownian particles move in a bath with constant large dissipation this adiabatic elimination of fast variables leads to the correct description of the system. However, in presence of hydrodynamic interaction, i.e.,

when the dissipation is position dependent or equivalently, when the noise is multiplicative with respect to system variables, the conventional adiabatic elimination of fast variables does not work. Using the method proposed by Sancho *et al.*¹⁷, which is based on a systematic expansion of the relevant variables in powers of Γ^{-1} and neglecting terms $\mathcal{O}(\Gamma^{-1})$, the Fokker-Planck equation corresponding to the Langevin equation (15) in the overdamped limit can be obtained as:

$$\begin{aligned} \frac{\partial F}{\partial t} = & \frac{\partial}{\partial x} \left\{ \frac{U'(x)}{\Gamma(x)} F \right\} + \gamma_a T_a \frac{\partial}{\partial x} \left\{ \frac{1}{\Gamma(x)} \frac{\partial}{\partial x} \frac{1}{\Gamma(x)} F \right\} \\ & + \gamma_b \bar{T}_b \frac{\partial}{\partial x} \left\{ \frac{f'(x)}{\Gamma(x)} \frac{\partial}{\partial x} \frac{f'(x)}{\Gamma(x)} F \right\} \\ & + \gamma_b \bar{T}_b \frac{\partial}{\partial x} \left\{ \frac{f'(x) f''(x)}{\Gamma^2(x)} F \right\} \end{aligned} \quad (16)$$

where $F(x, t)$ is the probability distribution function. In the ordinary Stratonovich description the Langevin equation corresponding to the Fokker-Planck equation (15) is given by

$$\dot{x} = -\frac{U'(x)}{\Gamma(x)} - \frac{f'(x) f''(x)}{\Gamma^2(x)} + \frac{1}{\Gamma(x)} \eta_A(t) + \frac{f'(x)}{\Gamma(x)} \eta_B(t) \quad (17)$$

Here, $\langle \eta_A \rangle = \langle \eta_B \rangle = 0$ and $\langle \eta_A(t) \eta_A(t') \rangle = 2\gamma_a T_a \delta(t-t')$, $\langle \eta_B(t) \eta_B(t') \rangle = 2\gamma_b \bar{T}_b \delta(t-t')$ and $\bar{T}_b = [T_b + (D_0 \kappa_0^2)]$ is the *effective temperature* of the bath $\{Q_j, P_j\}$ and we have set $k_B = 1$.

To compute the mean velocity, $\langle \dot{x} \rangle$, required to study the phenomena of fluctuation induced transport, let $U(x)$ and $f(x)$ be periodic functions of x and are invariant under the same transformation $x \rightarrow (x + L)$. Then, following Risken¹⁸, the exact mean velocity $\langle \dot{x} \rangle$ of the system is given by

$$\langle \dot{x} \rangle = \frac{1 - \exp(-\delta)}{\int_0^L dy \exp[-\phi(y)] \int_y^{y+L} dx' G(x') \exp[\phi(x')]} \quad (18)$$

with $\delta = \phi(x) - \phi(x + L)$ and $G(x) = \Gamma^2(x)/[T_a \gamma_a + \bar{T}_b \gamma_b f^2(x)]$. In the above expression $\phi(x)$ is the effective potential and is given by

$$\begin{aligned} \phi(x) = & \int^x \left\{ \frac{U'(x) \Gamma(x)}{T_a \gamma_a + \bar{T}_b \gamma_b [f'(x)]^2} \right. \\ & \left. + \frac{\bar{T}_b - T_a}{\Gamma(x)} \frac{2\gamma_a \gamma_b f'(x) f''(x)}{\gamma_a T_a + \gamma_b \bar{T}_b [f'(x)]^2} \right\} dx. \end{aligned} \quad (19)$$

From Eq.(19) it is clear that when $U'(x) = 0$ and $f'(x)$ as well as $f''(x)$ both have same sign, the direction of transport will depend on the relative sign of $(\bar{T}_b - T_a)$. For $\bar{T}_b > T_a$, the current will flow in one direction, on the other hand if $\bar{T}_b < T_a$, the direction is reverse. Also it is easy to verify from equations (18) that when $\bar{T}_b = T_a$ the current vanishes identically as $\delta = 0$ in this case. The nonvanishing δ makes $\phi(x)$ asymmetric with an effective slope which leads to the generation of directed motion.

We are now in a position to discuss some important issues related to our development which will indicate the relationship of our formalism with the existing ones in the same direction. It is important to note that the stationary distribution function associated with Eq.(16) is given by $F_S(x) = N \exp(-\phi(x))$, N being the normalization constant. It reduces to the correct equilibrium distribution under the situation $T_a = \bar{T}_b$. There will be no net current if $T_a = \bar{T}_b$, since in such case $\delta = 0$ which makes the numerator in Eq.(18) equals to zero. Earlier Millanos⁶ and Jayannavar⁷ has shown that when the system is coupled with two baths having different temperatures, a net current flows from higher to lower temperature. But in our case, this situation is quite different. If both the baths are initially kept at the same temperature $T_a = T_b = T$ (say), there will be current due to the presence of the term $(D_0 \kappa_0^2)$ in the expression of effective temperature \bar{T}_b which arises due to the modulation of nonlinear bath by external noise. If the two baths consist of the same type of oscillators and kept at the same equilibrium temperature, we can generate a current by externally driving the nonlinear baths. In such a case, the effective potential becomes

$$\begin{aligned} \phi(x) = & \int^x \left\{ \frac{U'(x) \Gamma(x)}{T \Gamma(x) + \gamma_b D_0 \kappa_0^2 [f'(x)]^2} \right. \\ & \left. + \frac{D_0 \kappa_0^2}{\Gamma(x)} \left(\frac{2\gamma_a \gamma_b f'(x) f''(x)}{T \Gamma(x) + \gamma_b D_0 \kappa_0^2 [f'(x)]^2} \right) \right\} dx. \end{aligned} \quad (20)$$

From the above expression it is clear that $\phi(x) \neq \phi(x + L)$ and even when the two baths are at the same temperature, there will be a current from $\{Q, P\}$ to $\{q, p\}$ bath. Let the coupling of the system with $\{Q, P\}$ be linear: $f(x) = x$ and hence $f''(x) = 0$, then the effective potential $\phi(x)$ satisfies the relation $\phi(2\pi) = \phi(0)$ and consequently, *there will be no net current*. As in the models of Millanos⁶ and Jayannavar⁷, one can easily demonstrate that there is no net current, if γ_a or γ_b are zero, as it should be. Finally, we would like to point out that the expression of current, Eq.(16) in our model reduces to the one obtained by Jayannavar, if there is no external modulation of the heat bath. At the end of this section we want to mentioned the fact that, instead of modulating the nonlinear bath, analogous situation would have been created by driving externally the linear heat bath and then one can obtain the corresponding expression for $\langle \dot{x} \rangle$ and $\phi(x)$.

B. Bath modulation by external colored noise

At this juncture, we consider the case where internal dissipation is Markovian but the external noise $\epsilon(t)$ is Gaussian, stationary, and exponentially correlated one with zero mean [i.e. $\langle \epsilon(t) \rangle = 0$ and $\langle \epsilon(t) \epsilon(t') \rangle = (D_e/\tau_e) \exp(-|t - t'|/\tau_e)$]. Consequently, neglecting the transient terms, the correlation function of the dressed noise $\pi(t)$ can be found to be¹³: $\langle \pi(t) \pi(t') \rangle =$

$(D_e \gamma_b \kappa_0^2 / \tau_e) \exp(-|t - t'| / \tau_e)$. Thus the effective noise $\eta(t)$ will also be exponentially correlated: $\langle \langle \eta(t) \rangle \rangle_b = 0$ and $\langle \langle \eta(t) \eta(t') \rangle \rangle_b = (D_R / \tau_R) \exp(-|t - t'| / \tau_R)$ where $D_R = \gamma_b (T_b + D_e \kappa_0^2)$ and $\tau_R = (\gamma_b \kappa_0^2 D_e \tau_e) / D_R$. For $\tau_e \rightarrow 0$ we obtain the previous case of δ -correlated noise process. Now using van-Kampen's approach¹⁹, the Fokker-Planck equation in phase space corresponding to Langevin equation (3) becomes

$$\begin{aligned} \frac{\partial F}{\partial t} = & -\frac{\partial}{\partial x}(vF) \\ & + \frac{\partial}{\partial v}[\Gamma(x)v + U'(x) - 2f'(x)f''(x)\tau_R D_R]F \\ & + [f'(x)]^2 \tau_R D_R \frac{\partial^2 F}{\partial x \partial v} \\ & + \{[f'(x)]^2 D_R - \Gamma(x)[f'(x)]^2 \tau_R D_R\} \frac{\partial^2 F}{\partial v^2} \end{aligned} \quad (21)$$

The above equation is valid for small but finite correlation time τ_e . The term $[f'(x)]^2 \tau_R D_R (\partial^2 F / \partial x \partial v)$ is a small non-Markovian contribution and for small τ_R , we may neglect this term to get finally the approximate Fokker-Planck equation in phase space,

$$\begin{aligned} \frac{\partial F}{\partial t} = & -\frac{\partial}{\partial x}(vF) \\ & + \frac{\partial}{\partial v}[\Gamma(x)v + U'(x) - 2f'(x)f''(x)\tau_R D_R]F \\ & + \{[f'(x)]^2 D_R - \Gamma(x)[f'(x)]^2 \tau_R D_R\} \frac{\partial^2 F}{\partial v^2} \end{aligned} \quad (22)$$

which can equivalently be described by the Langevin equation in the ordinary Stratonovich sense:

$$\dot{x} = v, \dot{v} = -U'(x) - \Gamma(x)v + g(x)\eta_e(t) \quad (23)$$

where $g(x) = f'(x)[1 - \Gamma(x)\tau_R]^{1/2}$ and η_e is a Gaussian noise with zero mean and $\langle \eta_e(t)\eta_e(t') \rangle = 2D_R\delta(t - t')$. Along the same line as in the first case of δ -correlated noises, the expression for the average velocity may be calculated from Eq.(18) by replacing the effective potential $\phi(x)$ by the new effective potential $\psi(x)$ given by

$$\begin{aligned} \psi(x) = & \int^x \left\{ \frac{U'(x)\Gamma(x)}{T_a\gamma_a + \bar{T}_b\gamma_b g^2(x)} \right. \\ & \left. + \frac{\bar{T}_b - T_a}{\Gamma(x)} \left(\frac{2\gamma_a\gamma_b g(x)g'(x)}{\gamma_a T_a + \gamma_b \bar{T}_b g^2(x)} \right) \right\} dx. \end{aligned} \quad (24)$$

When both the baths, A and B, have same temperature, i.e. for $T_a = T_b = T$, $\psi(x)$ reduces to

$$\begin{aligned} \psi(x) = & \int^x \left\{ \frac{U'(x)\Gamma(x)}{T\gamma_a + T\gamma_b g^2(x) + \gamma_b D_R \kappa_0^2 g^2(x)} \right. \\ & \left. + \frac{D_R \kappa_0^2}{\Gamma(x)} \left(\frac{2\gamma_a\gamma_b g(x)g'(x)}{\gamma_a T + \gamma_b T g^2(x) + \gamma_b D_R \kappa_0^2 g^2(x)} \right) \right\} dx. \end{aligned} \quad (25)$$

Here also for linear coupling, i.e. for $f(x) = x$, there would be no net current.

IV. GENERAL ANALYSIS

Two interesting points may be noted here. For $U(x) = 0$, i.e., even in the absence of any external potential, fluctuation induced directed motion is possible. The direction of current will be from nonlinear bath to linear bath for $T_b = T_a = T$ (say). Thus, the system will act like a Carnot engine, which extracts work by making use of two thermal baths not necessarily at different temperatures. Even when both the baths are kept at $T = 0$, this engine operates due to external modulation of the nonlinear bath which for δ -correlated external noise modulation, operates between two temperatures 0 and $D_0 \kappa_0^2$. On the other hand, in the case of exponentially correlated external noise, the engine operates between 0 and D_R where $D_R = D_e \kappa_0^2$. At this point it is pertinent to mention that instead of mentioning the physical temperature, the effective temperature induced by external driving is the relevant measure. From the foregoing discussion it is evident that the model shows the net motion occurs as long as the *relative effective temperature* difference between the two baths (A and B) is nonzero.

From the very mode of development it is clear that the magnitude of the net current will depend on the slope of the effective potentials $\phi(x)$ and $\psi(x)$. For $T = 0$ and $U(x) = 0$, the effective slope of $\phi(x)$ is $(2f''(x)\gamma_a)/(\Gamma(x)f'(x))$, whereas that for $\psi(x)$ is

$$\begin{aligned} \psi(x) = & \frac{2f''(x)\gamma_a}{\Gamma(x)f'(x)} - \frac{\tau_R \Gamma'(x)\gamma_a}{\Gamma(x)[1 - \Gamma(x)\tau_R]} \\ \approx & \frac{2f''(x)\gamma_a}{\Gamma(x)f'(x)} - \frac{\gamma_a \tau_R \Gamma'(x)}{\Gamma(x)}. \end{aligned} \quad (26)$$

In the last step we have used the fact that for $\tau_R \Gamma(x) < 1$.

When both $\Gamma(x)$ and $\Gamma'(x)$ are positive (and $\Gamma(x)\tau_R < 1$), the slope of the effective potential $\psi(x)$ is less than that of $\phi(x)$ and consequently the net flow reduces. Thus the magnitude of transport will be maximum if we modulate the nonlinear bath by δ -correlated noise. Any finite correlation time will decrease the transport process for $\Gamma(x) > 0$ and $\Gamma'(x) > 0$. On the other hand for $\Gamma'(x)$ negative, the slope of $\psi(x)$ becomes larger which increases the net flow. Thus for a given correlation time, the magnitude of current will primarily depend on the relative sign of $\Gamma(x)$ and $\Gamma'(x)$. Also for a fixed value of coupling function $f(x)$, the ratio of γ_b and γ_a determines the magnitude of the current. On the other hand when the amplitude modulations of $f'(x)$ and $f''(x)$ are small in comparison to the amplitude modulation of $U(x)$, the slope of the effective potential ϕ will be approximately given by $(U'(x)\Gamma(x))/(\gamma_b D_0 \kappa_0^2 [f'(x)]^2)$ at $T = 0$ and that of ψ will be given by $(U'(x)\Gamma(x))/(\gamma_b D_R \kappa_0^2 g^2(x)) \approx$

$((U'(x)\Gamma(x))/(\gamma_b D_R \kappa [f'(x)]^2))[1 + \Gamma(x)\tau_R]$. This expression provides an interesting fact that when the amplitude modulation of $f'(x)$ and $f''(x)$ are small, the net current increases if we modulate the bath by exponentially correlated noise. In this particular limit, the problem becomes equivalent to a particle moving in a spatially varying temperature field, namely, $T(x) = T\Gamma(x) + \gamma_b D_0 \kappa_0^2 [f'(x)]^2$. Almost three decades ago Landauer²⁰ explored the problem of characterizing non-equilibrium steady states in the transition kinetics between the two locally stable bistable systems. His idea was that the relative stability of a particle diffusing in a bistable potential can be altered by an intervening hot layer which has the effect of pumping particles from a globally stable region to a metastable region. Latter this problem was carried in a wider context by van Kampen and others²¹. In absence of any externally applied fields, Büttiker²² suggested that the generation of current is an important consequence of state dependent diffusion. In contrast, we address the problem of Langevin equation with multiplicative noise and state dependent diffusion for a thermodynamically open system to study the non-equilibrium fluctuation induced transport phenomena.

V. CONCLUSIONS

In this article we have developed a microscopic model of a Maxwell demon type engine. Our approach is based

on the system-reservoir model where the system is coupled with two baths. For one bath, the system-reservoir coupling is bi-linear and for the other, the coupling is non-linear in system coordinate which is externally modulated by a noise agency. We then derive the Langevin equation with a multiplicative noise and a nonlinear dissipation. Then by a systematic expansion of the relevant variable in powers of inverse of dissipation constant, we obtained the corresponding Smoluchowski equation for state dependent diffusion in the limit of large friction. We have applied the formalism to the problem of diffusion of a particle in a periodic potential where the nonlinear bath act as the source with an *effective temperature* and the linear bath as the sink and there by providing a heat engine.

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